By orthogonal principle, we have

\[ S = R_{sr} R_{rr}^{-1}. \]

Since \( r = C_{circ}s + q \), we have \( R_{rr} = \varepsilon_s C_{circ} C_{circ}^\dagger + N_0 I_M \) and \( R_{sr} = \varepsilon_s C_{circ}^\dagger \).

Using \( C_{circ} = W^\dagger \Gamma W \), we can obtain

\[ S = W^\dagger \varepsilon_s \Gamma W (\varepsilon_s \Gamma \Gamma^\dagger + N_0 I_M)^{-1} W = W^\dagger D W \tag{1} \]

where \( D \) is a diagonal matrix and \( D_{kk} = \frac{\gamma C_k^*}{1 + \gamma |C_k|^2} \).

6.10

(a)

By (6.21), the overall transfer function is

\[ T = W^\dagger \Lambda \Gamma W, \]

where \( \Lambda \Gamma \) is diagonal and \( k \)th diagonal element of it is \( \frac{\gamma |C_k|^2}{1 + \gamma |C_k|^2} \).

Therefore, \( T \) is circulant and the \( t_{ii} \) is

\[ t_{ii} = \frac{1}{M} \sum_{n=0}^{M-1} \frac{\gamma |C_k|^2}{1 + \gamma |C_k|^2}. \]

(b)

Using (3.18), we have

\[ \beta(i) = \frac{t_{ii}}{1 - t_{ii}} = \frac{\sum_{k=0}^{M-1} \frac{\gamma |C_k|^2}{1 + \gamma |C_k|^2}}{M - \sum_{k=0}^{M-1} \frac{1}{1 + \gamma |C_k|^2}}. \]

Using (3.11), we have

\[ \beta_{biased}(i) = \frac{1}{1 - t_{ii}} = \frac{M}{\sum_{k=0}^{M-1} \frac{1}{1 + \gamma |C_k|^2}}. \]

(c)

Using (3.20),

\[ \sigma_{e_i}^2 = (1 - t_{ii}) \varepsilon_s = \frac{\varepsilon_s}{M} \sum_{k=0}^{M-1} \frac{1}{1 + \gamma |C_k|^2}. \]

6.11

For the CP OFDM, \( \beta_{ofdm}(k) = \gamma |C_k|^2 \).

For SC-CP with ZF receiver,

\[ \beta_{sc-cp} = \frac{1}{M} \sum_{l=0}^{M-1} \frac{1}{\gamma |C_l|^2}. \]

For SC-CP with MMSE receiver,

\[ \beta = \frac{1}{M} \sum_{l=0}^{M-1} \frac{1}{1 + \gamma |C_l|^2} - 1. \]
The performance of those systems depend on $|C_k|^2$ for all $k$.
$|C_k|^2 = |C(e^{j2\pi/k})|^2$, so the systems performance are affected by the magnitude response of $C(z)$.

From (6.25) $S = W^\dagger \Gamma^{-1} W \Upsilon$, the output error vector $e$ can be expressed as

$$e = W^\dagger \Gamma^{-1} W \Upsilon q.$$ 

The autocorrelation matrix of the output error $R_e = E[ee^\dagger]$ is

$$R_e = E[W^\dagger \Gamma^{-1} W \Upsilon q q^\dagger \Upsilon^\dagger W^\dagger \Gamma^{-1} W] = N_0 W^\dagger \Gamma^{-1} W \Upsilon \Upsilon^\dagger W^\dagger \Gamma^{-1} W = N_0 W^\dagger \Gamma^{-1} W D W^\dagger \Gamma^{-1} W,$$

where $D = \begin{bmatrix} 2I_\nu & 0 \\ 0 & I_{M-\nu} \end{bmatrix}$.

The subchannel noise variance

$$\sigma_{e_k}^2 = [R_e]_{kk} = [N_0 W^\dagger \Gamma^{-1} W D W^\dagger \Gamma^{-1} W]_{kk}.$$