2.5

(a)

\[ x_a(t) = \left[ \sum_{n=0}^{5} x(n) \delta(t - nT) \right] \ast p_1(t) \]

\[ w_a(t) = p_2(t) \ast c_a(t) \ast x_a(t) = p_2(t) \ast x_a(t) \]

(b)

\[ r(n) = w_a(t) \bigg|_{t = nT} = 0.5\delta(n - 1) + 0.5\delta(n - 2) - 0.5\delta(n - 7) - 0.5\delta(n - 8). \]

(c)

\[ c(n) = \left[ p_2(t) \ast c_a(t) \ast p_1(t) \right] \bigg|_{t = nT} = 0.5\delta(n - 1) + \delta(n - 2) + 0.5\delta(n - 3). \]

\[ r(n) = c(n) \ast x(n) = 0.5\delta(n - 1) + 0.5\delta(n - 2) - 0.5\delta(n - 7) - 0.5\delta(n - 8). \]

2.6

(a)

Because \( q_a(t) \) is a WSS process and \( p_2(t) \) is LTI, the noise

\[ w_a(t) = p_2(t) \ast q_a(t) \]

is also a WSS process. The discrete noise is

\[ q(n) = w_a(t) \bigg|_{t = nT}. \]
The expectation of \( q(n) \) is

\[
E[q(n)] = E[w_a(nT)] = \text{constant}.
\]

The auto-correlation of \( q(n) \) is

\[
E[q(n)q(m)^*] = E[w_a(nT)w_a(mT)^*] = R_{w_a}((n-m)T),
\]

which depends only on the difference \( n-m \). Therefore, \( q(n) \) is a WSS process. The PSD of \( w_a(t) \) is

\[
S_{w_a}(f) = |P_2(f)|^2S_{q_a}(f) = \begin{cases} N_a, & -0.5/T \leq f \leq 0.5/T; \\ 0, & \text{otherwise}. \end{cases}
\]

The auto-correlation of \( w_a(t) \) is

\[
R_{w_a}(\tau) = \frac{N_a}{T} \text{sinc}(\frac{\tau}{T}).
\]

Hence the auto-correlation of \( q(n) \) is given by

\[
R_q(k) = R_{w_a}(\tau) \Big|_{\tau=kt} = \begin{cases} \frac{N_a}{T}, & k = 0; \\ 0, & \text{otherwise}. \end{cases}
\]

Therefore \( q(n) \) is white and the noise power is \( R_q(0) = \frac{N_a}{T} \).

(b)

Similarly as (a), \( q(n) \) is a WSS process. The PSD of \( w_a(t) \) is

\[
S_{w_a}(f) = |P_2(f)|^2S_{q_a}(f) = \begin{cases} N_a, & -1/T \leq f \leq 1/T; \\ 0, & \text{otherwise}. \end{cases}
\]

The auto-correlation of \( w_a(t) \) is

\[
R_{w_a}(\tau) = \frac{2N_a}{T} \text{sinc}(\frac{2\tau}{T}).
\]

Hence the auto-correlation of \( q(n) \) is given by

\[
R_q(k) = R_{w_a}(\tau) \big|_{\tau=kt} = \begin{cases} \frac{2N_a}{T}, & k = 0; \\ 0, & \text{otherwise}. \end{cases}
\]

Therefore \( q(n) \) is white and the noise power is \( R_q(0) = \frac{2N_a}{T} \).

2.7

In a 2-bit PAM, a codeword is mapped to \( s = \pm \Delta, \pm 3\Delta \).

Since \( r(n) = \alpha s(n) \) where \( 0 < \alpha \leq 1 \), the received symbols \( \hat{s} = \pm \alpha \Delta, \pm 3\alpha \Delta \).

For \( \hat{s} = \pm \alpha \Delta, e = |\hat{s} - s| = |\alpha - 1|\Delta < \Delta \Rightarrow P(\hat{s} \neq s|s = \pm \alpha) = 0 \),

\[
\hat{s} = \pm 3\alpha \Delta, e = |\hat{s} - s| = |3\alpha - 1|\Delta = \begin{cases} \Delta, & \text{if } 0 < \alpha < \frac{2}{3} \\ < \Delta, & \text{if } \frac{2}{3} < \alpha < 1 \end{cases}
\]

\[
\Rightarrow P(\hat{s} \neq s|s = \pm \alpha) = \begin{cases} 1, & \text{if } 0 < \alpha < \frac{2}{3} \\ 0, & \text{if } \frac{2}{3} < \alpha < 1 \end{cases}
\]

\[
SER_{\text{average}} = \begin{cases} \frac{1}{2}, & \text{if } 0 < \alpha < \frac{2}{3} \\ 0, & \text{if } \frac{2}{3} < \alpha < 1 \end{cases}
\]
For each subchannel, \( \sigma_{e_i}^2 = \frac{N_i}{|\alpha_i|^2} \), \( SNR_i = \frac{e_i}{\sigma_{e_i}^2} \), \( BER_i = Q(\sqrt{SNR_i}) \)

0-th subchannel, \( \sigma_{e_0}^2 = \frac{10}{5^2} = 0.4 \), \( SNR_0 = \frac{40}{0.4} = 25 \), \( BER_0 = Q(5) \)

1-st subchannel, \( \sigma_{e_1}^2 = \frac{10}{17} = 10 \), \( SNR_1 = \frac{10}{10} = 1 \), \( BER_1 = Q(1) \)

2-nd subchannel, \( \sigma_{e_2}^2 = \frac{10}{100} = 0.1 \), \( SNR_2 = \frac{10}{0.1} = 100 \), \( BER_2 = Q(10) \)

3-rd subchannel, \( \sigma_{e_3}^2 = \frac{10}{\sqrt{40}} = 0.25 \), \( SNR_3 = \frac{10}{0.25} = 40 \), \( BER_3 = Q(\sqrt{40}) \)

The average BER is

\[
P_{e,avg} = \frac{1}{4} [Q(5) + Q(1) + Q(10) + Q(\sqrt{40})]
\]

For PAM symbols, \( SER(b) = 2(1 - 2^{-b})Q(\sqrt{\frac{3}{2^{2b-1}}SNR}) \)

\( SER_0 = SER_1 = 0 \)

\( SER_2 = 2(1 - 2^{-2})Q(\sqrt{\frac{3}{2^{2-1}}100}) = 5.81 \times 10^{-6} \)

\( SER_3 = 2(1 - 2^{-2})Q(\sqrt{\frac{3}{2^{2-1}}40}) = 3.5 \times 10^{-3} \)

\( SER_{average} = \frac{1}{4} \sum_{i=0}^{3} SER_i = 8.76 \times 10^{-4} \)

\( BER_{average} = \frac{1}{2} SER_{average} = 4.38 \times 10^{-4} \)